

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Equivalent expressions	■ ■ ■

ID: 371cbf6b

$$(ax+3)(5x^2-bx+4)=20x^3-9x^2-2x+12$$

The equation above is true for all x , where a and b are constants. What is the value of ab ?

- A. 18
- B. 20
- C. 24
- D. 40

ID: 371cbf6b Answer

Correct Answer: C

Rationale

Choice C is correct. If the equation is true for all x , then the expressions on both sides of the equation will be equivalent. Multiplying the polynomials on the left-hand side of the equation gives $5ax^3 - abx^2 + 4ax + 15x^2 - 3bx + 12$. On the right-hand side of the equation, the only x^2 -term is $-9x^2$. Since the expressions on both sides of the equation are equivalent, it follows that $-abx^2 + 15x^2 = -9x^2$, which can be rewritten as $(-ab + 15)x^2 = -9x^2$. Therefore, $-ab + 15 = -9$, which gives $ab = 24$.

Choice A is incorrect. If $ab = 18$, then the coefficient of x^2 on the left-hand side of the equation would be $-18 + 15 = -3$, which doesn't equal the coefficient of x^2 , -9 , on the right-hand side. Choice B is incorrect. If $ab = 20$, then the coefficient of x^2 on the left-hand side of the equation would be $-20 + 15 = -5$, which doesn't equal the coefficient of x^2 , -9 , on the right-hand side. Choice D is incorrect. If $ab = 40$, then the coefficient of x^2 on the left-hand side of the equation would be $-40 + 15 = -25$, which doesn't equal the coefficient of x^2 , -9 , on the right-hand side.

Question Difficulty: Hard

Assessment	Test	Domain	Skill	Difficulty
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ID: 40c09d66

$\frac{\sqrt{x^5}}{\sqrt[3]{x^4}} = x^{\frac{a}{b}}$
If $\frac{\sqrt{x^5}}{\sqrt[3]{x^4}} = x^{\frac{a}{b}}$ for all positive values of x ,
what is the value of $\frac{a}{b}$?

ID: 40c09d66 Answer

Rationale

The correct answer is $\frac{7}{6}$. The value of $\frac{a}{b}$ can be found by first rewriting the left-hand side of the given equation as $x^{\frac{\frac{5}{2}}{\frac{4}{3}}}$. Using the properties of exponents, this expression can be rewritten as $x^{\left(\frac{5}{2} - \frac{4}{3}\right)}$. This expression can be rewritten by subtracting the fractions in the exponent, which yields $x^{\frac{7}{6}}$. Thus, $\frac{a}{b}$ is $\frac{7}{6}$. Note that 7/6, 1.166, and 1.167 are examples of ways to enter a correct answer.

Question Difficulty: Hard

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ID: 34847f8a

$$\frac{2}{x-2} + \frac{3}{x+5} = \frac{rx+t}{(x-2)(x+5)}$$

The equation above is true for all $x > 2$, where r and t are positive constants. What is the value of rt ?

- A. -20
- B. 15
- C. 20
- D. 60

ID: 34847f8a Answer

Correct Answer: C

Rationale

Choice C is correct. To express the sum of the two rational expressions on the left-hand side of the equation as the single rational expression on the right-hand side of the equation, the expressions on the left-hand side must have the same denominator. Multiplying the first expression by $\frac{x+5}{x-5}$ results in $\frac{2(x+5)}{(x-2)(x+5)}$, and multiplying the second expression by $\frac{x-2}{x-2}$ results in $\frac{3(x-2)}{(x-2)(x+5)}$, so the given equation can be rewritten as $\frac{2(x+5)}{(x-2)(x+5)} + \frac{3(x-2)}{(x-2)(x+5)} = \frac{rx+t}{(x-2)(x+5)}$, or $\frac{2x+10}{(x-2)(x+5)} + \frac{3x-6}{(x-2)(x+5)} = \frac{rx+t}{(x-2)(x+5)}$. Since the two rational expressions on the left-hand side of the equation have the same denominator as the rational expression on the right-hand side of the equation, it follows that $(2x+10) + (3x-6) = rx+t$. Combining like terms on the left-hand side yields $5x+4 = rx+t$, so it follows that $r=5$ and $t=4$. Therefore, the value of rt is $(5)(4) = 20$.

Choice A is incorrect and may result from an error when determining the sign of either r or t . Choice B is incorrect and may result from not distributing the 2 and 3 to their respective terms in

$\frac{2(x+5)}{(x-2)(x+5)} + \frac{3(x-2)}{(x-2)(x+5)} = \frac{rx+t}{(x-2)(x+5)}$. Choice D is incorrect and may result from a calculation error.

Question Difficulty: Hard

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ID: 137cc6fd

$$\sqrt[5]{70n}\left(\sqrt[6]{70n}\right)^2$$

For what value of x is the given expression equivalent to $(70n)^{30x}$, where $n > 1$?

ID: 137cc6fd Answer

Correct Answer: .0177, .0178, 4/225

Rationale

The correct answer is $\frac{4}{225}$. An expression of the form $\sqrt[k]{a}$, where k is an integer greater than 1 and $a \geq 0$, is equivalent to $a^{\frac{1}{k}}$. Therefore, the given expression, where $n > 1$, is equivalent to $(70n)^{\frac{1}{5}}\left((70n)^{\frac{1}{6}}\right)^2$. Applying properties of exponents, this expression can be rewritten as $(70n)^{\frac{1}{5}}(70n)^{\frac{1}{6}\cdot 2}$, or $(70n)^{\frac{1}{5}}(70n)^{\frac{1}{3}}$, which can be rewritten as $(70n)^{\frac{1}{5}+\frac{1}{3}}$, or $(70n)^{\frac{8}{15}}$. It's given that the expression $\sqrt[5]{70n}\left(\sqrt[6]{70n}\right)^2$ is equivalent to $(70n)^{30x}$, where $n > 1$. It follows that $(70n)^{\frac{8}{15}}$ is equivalent to $(70n)^{30x}$. Therefore, $\frac{8}{15} = 30x$. Dividing both sides of this equation by 30 yields $\frac{8}{450} = x$, or $\frac{4}{225} = x$. Thus, the value of x for which the given expression is equivalent to $(70n)^{30x}$, where $n > 1$, is $\frac{4}{225}$. Note that 4/225, .0177, .0178, 0.017, and 0.018 are examples of ways to enter a correct answer.

Question Difficulty: Hard

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Equivalent expressions	<div><div></div><div></div><div></div></div>

ID: ea6d05bb

The expression $(3x - 23)(19x + 6)$ is equivalent to the expression $ax^2 + bx + c$, where a , b , and c are constants. What is the value of b ?

ID: ea6d05bb Answer

Correct Answer: -419

Rationale

The correct answer is -419 . It's given that the expression $(3x - 23)(19x + 6)$ is equivalent to the expression $ax^2 + bx + c$, where a , b , and c are constants. Applying the distributive property to the given expression, $(3x - 23)(19x + 6)$, yields $(3x)(19x) + (3x)(6) - (23)(19x) - (23)(6)$, which can be rewritten as $57x^2 + 18x - 437x - 138$. Combining like terms yields $57x^2 - 419x - 138$. Since this expression is equivalent to $ax^2 + bx + c$, it follows that the value of b is -419 .

Question Difficulty: Hard

Assessment	Test	Domain	Skill	Difficulty
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ID: d8789a4c

$$\frac{x^2 - c}{x - b}$$

In the expression above, b and c are positive integers. If the expression is equivalent to $x + b$ and $x \neq b$, which of the following could be the value of c ?

- A. 4
- B. 6
- C. 8
- D. 10

ID: d8789a4c Answer

Correct Answer: A

Rationale


Choice A is correct. If the given expression is equivalent to $x + b$, then $\frac{x^2 - c}{x - b} = x + b$, where x isn't equal to b . Multiplying both sides of this equation by $x - b$ yields $x^2 - c = (x + b)(x - b)$. Since the right-hand side of this equation is in factored form for the difference of squares, the value of c must be a perfect square. Only choice A gives a perfect square for the value of c .

Choices B, C, and D are incorrect. None of these values of c produces a difference of squares. For example,

when 6 is substituted for c in the given expression, the result is $\frac{x^2 - 6}{x - b}$. The expression $x^2 - 6$ can't be

factored with integer values, and therefore $\frac{x^2 - 6}{x - b}$ isn't equivalent to $x + b$.

Question Difficulty: Hard

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Equivalent expressions	

ID: 5355c0ef

$$0.36x^2 + 0.63x + 1.17$$

The given expression can be rewritten as $a(4x^2 + 7x + 13)$, where a is a constant. What is the value of a ?

ID: 5355c0ef Answer

Correct Answer: .09, 9/100

Rationale

The correct answer is .09. It's given that the expression $0.36x^2 + 0.63x + 1.17$ can be rewritten as $a(4x^2 + 7x + 13)$. Applying the distributive property to the expression $a(4x^2 + 7x + 13)$ yields $4ax^2 + 7ax + 13a$. Therefore, $0.36x^2 + 0.63x + 1.17$ can be rewritten as $4ax^2 + 7ax + 13a$. It follows that in the expressions $0.36x^2 + 0.63x + 1.17$ and $4ax^2 + 7ax + 13a$, the coefficients of x^2 are equivalent, the coefficients of x are equivalent, and the constant terms are equivalent. Therefore, $0.36 = 4a$, $0.63 = 7a$, and $1.17 = 13a$. Solving any of these equations for a yields the value of a . Dividing both sides of the equation $0.36 = 4a$ by 4 yields $0.09 = a$. Therefore, the value of a is 0.09. Note that .09 and 9/100 are examples of ways to enter a correct answer.

Question Difficulty: Hard

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Equivalent expressions	■ ■ ■

ID: c81b6c57

In the expression $3(2x^2 + px + 8) - 16x(p + 4)$, p is a constant. This expression is equivalent to the expression $6x^2 - 155x + 24$. What is the value of p ?

- A. -3
- B. 7
- C. 13
- D. 155

ID: c81b6c57 Answer

Correct Answer: B

Rationale

Choice B is correct. Using the distributive property, the first given expression can be rewritten as $6x^2 + 3px + 24 - 16px - 64x + 24$, and then rewritten as $6x^2 + (3p - 16p - 64)x + 24$. Since the expression $6x^2 + (3p - 16p - 64)x + 24$ is equivalent to $6x^2 - 155x + 24$, the coefficients of the x terms from each expression are equivalent to each other; thus $3p - 16p - 64 = -155$. Combining like terms gives $-13p - 64 = -155$. Adding 64 to both sides of the equation gives $-13p = -91$. Dividing both sides of the equation by -13 yields $p = 7$.

Choice A is incorrect. If $p = -3$, then the first expression would be equivalent to $6x^2 - 25x + 24$. Choice C is incorrect. If $p = 13$, then the first expression would be equivalent to $6x^2 - 233x + 24$. Choice D is incorrect. If $p = 155$, then the first expression would be equivalent to $6x^2 - 2,079x + 24$.

Question Difficulty: Hard

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Equivalent expressions	<div><div></div><div></div><div></div></div>

ID: 2c88af4d

$$\frac{x^{-2}y^{\frac{1}{2}}}{x^{\frac{1}{3}}y^{-1}}$$

The expression $x^{\frac{1}{3}}y^{-1}$, where $x > 1$ and $y > 1$, is equivalent to which of the following?

- $\frac{\sqrt{y}}{\sqrt[3]{x^2}}$

A.
- $\frac{y\sqrt{y}}{\sqrt[3]{x^2}}$

B.
- $\frac{y\sqrt{y}}{x\sqrt{x}}$

C.
- $\frac{y\sqrt{y}}{x^2\sqrt[3]{x}}$

D.

ID: 2c88af4d Answer

Correct Answer: D

Rationale

Choice D is correct. For $x > 1$ and $y > 1$, $x^{\frac{1}{3}}$ and $y^{\frac{1}{2}}$ are equivalent to $\sqrt[3]{x}$ and \sqrt{y} , respectively. Also, x^{-2} and y^{-1} are equivalent to $\frac{1}{x^2}$ and $\frac{1}{y}$, respectively. Therefore, the given expression can be rewritten as $\frac{y\sqrt{y}}{x^2\sqrt[3]{x}}$.

Choices A, B, and C are incorrect because these choices are not equivalent to the given expression for $x > 1$ and $y > 1$.

For example, for $x = 2$ and $y = 2$, the value of the given expression is $2^{-\frac{5}{6}}$; the values of the choices, however, are $2^{-\frac{1}{3}}$, $2^{\frac{5}{6}}$, and 1, respectively.

Question Difficulty: Hard

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Equivalent expressions	■ ■ ■

ID: 22fd3e1f

$$f(x) = x^3 - 9x$$

$$g(x) = x^2 - 2x - 3$$

Which of the following expressions is

equivalent to $\frac{f(x)}{g(x)}$, for $x > 3$?

A. $\frac{1}{x+1}$

B. $\frac{x+3}{x+1}$

C. $\frac{x(x-3)}{x+1}$

D. $\frac{x(x+3)}{x+1}$

ID: 22fd3e1f Answer

Correct Answer: D

Rationale

Choice D is correct. Since $x^3 - 9x = x(x+3)(x-3)$ and $x^2 - 2x - 3 = (x+1)(x-3)$, the fraction $\frac{f(x)}{g(x)}$ can be written as $\frac{x(x+3)(x-3)}{(x+1)(x-3)}$. It is given that $x > 3$, so the common factor $x - 3$ is not equal to 0. Therefore, the fraction can be further simplified to $\frac{x(x+3)}{x+1}$.

Choice A is incorrect. The expression $\frac{1}{x+1}$ is not equivalent to $\frac{f(x)}{g(x)}$ because at $x = 0$, $\frac{1}{x+1}$ has a value of 1 and $\frac{f(x)}{g(x)}$ has a value of 0.

Choice B is incorrect and results from omitting the factor x in the factorization of $f(x)$. Choice C is incorrect and may result from incorrectly factoring $g(x)$ as $(x+1)(x+3)$ instead of $(x+1)(x-3)$.

Question Difficulty: Hard

Question ID a0b4103e

3.11

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Equivalent expressions	■ ■ ■

ID: a0b4103e

The expression $\frac{1}{3}x^2 - 2$ can be rewritten as $\frac{1}{3}(x - k)(x + k)$, where k is a positive constant. What is the value of k ?

- A. 2
- B. 6
- C. $\sqrt{2}$
- D. $\sqrt{6}$

ID: a0b4103e Answer

Correct Answer: D

Rationale

Choice D is correct. Factoring out the coefficient $\frac{1}{3}$, the given expression can be rewritten as $\frac{1}{3}(x^2 - 6)$. The expression $x^2 - 6$ can be approached as a difference of squares and rewritten as $(x - \sqrt{6})(x + \sqrt{6})$. Therefore, k must be $\sqrt{6}$.

Choice A is incorrect. If k were 2, then the expression given would be rewritten as $\frac{1}{3}(x - 2)(x + 2)$, which is equivalent to $\frac{1}{3}x^2 - \frac{4}{3}$, not $\frac{1}{3}x^2 - 2$.

Choice B is incorrect. This may result from incorrectly factoring the expression and finding $(x - 6)(x + 6)$ as the factored form of the expression. Choice C is incorrect. This may result from incorrectly distributing the $\frac{1}{3}$ and rewriting the expression as $\frac{1}{3}(x^2 - 2)$.

Question Difficulty: Hard

Question ID ad038c19

3.12

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Equivalent expressions	■ ■ ■

ID: ad038c19

Which of the following is

equivalent to $\left(a + \frac{b}{2}\right)^2$?

A. $a^2 + \frac{b^2}{2}$

B. $a^2 + \frac{b^2}{4}$

C. $a^2 + \frac{ab}{2} + \frac{b^2}{2}$

D. $a^2 + ab + \frac{b^2}{4}$

ID: ad038c19 Answer

Correct Answer: D

Rationale

Choice D is correct. The expression $\left(a + \frac{b}{2}\right)^2$ can be rewritten as $\left(a + \frac{b}{2}\right)\left(a + \frac{b}{2}\right)$. Using the distributive property, the expression yields $\left(a + \frac{b}{2}\right)\left(a + \frac{b}{2}\right) = a^2 + \frac{ab}{2} + \frac{ab}{2} + \frac{b^2}{4}$. Combining like terms gives $a^2 + ab + \frac{b^2}{4}$.

Choices A, B, and C are incorrect and may result from errors using the distributive property on the given expression or combining like terms.

Question Difficulty: Hard

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Equivalent expressions	■ ■ ■

ID: 12e7faf8

The equation $\frac{x^2+6x-7}{x+7} = ax+d$ is true for all $x \neq -7$, where a and d are integers. What is the value of $a+d$?

- A. -6
- B. -1
- C. 0
- D. 1

ID: 12e7faf8 Answer

Correct Answer: C

Rationale

Choice C is correct. Since the expression x^2+6x-7 can be factored as $(x+7)(x-1)$, the given equation can be rewritten as $\frac{(x+7)(x-1)}{x+7} = ax+d$. Since $x \neq -7$, $x+7$ is also not equal to 0, so both the numerator and denominator of $\frac{(x+7)(x-1)}{x+7}$ can be divided by $x+7$. This gives $x-1 = ax+d$. Equating the coefficient of x on each side of the equation gives $a = 1$. Equating the constant terms gives $d = -1$. The sum is $1 + (-1) = 0$.

Choice A is incorrect and may result from incorrectly simplifying the equation. Choices B and D are incorrect. They are the values of d and a , respectively, not $a+d$.

Question Difficulty: Hard

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Equivalent expressions	■ ■ ■

ID: 89fc23af

Which of the following expressions is

equivalent to $\frac{x^2 - 2x - 5}{x - 3}$?

A. $x - 5 - \frac{20}{x - 3}$

B. $x - 5 - \frac{10}{x - 3}$

C. $x + 1 - \frac{8}{x - 3}$

D. $x + 1 - \frac{2}{x - 3}$

ID: 89fc23af Answer

Correct Answer: D

Rationale

Choice D is correct. The numerator of the given expression can be rewritten in terms of the denominator, $x - 3$, as follows: $x^2 - 2x - 5 = x^2 - 3x + x - 3 - 2$, which is equivalent to $x(x - 3) + (x - 3) - 2$. So the given

expression is equivalent to $\frac{x(x - 3) + (x - 3) - 2}{x - 3} = \frac{x(x - 3)}{x - 3} + \frac{x - 3}{x - 3} - \frac{2}{x - 3}$. Since the given expression is

defined for $x \neq 3$, the expression can be rewritten as $x + 1 - \frac{2}{x - 3}$.

Long division can also be used as an alternate approach. Choices A, B, and C are incorrect and may result from errors made when dividing the two polynomials or making use of structure.

Question Difficulty: Hard

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Equivalent expressions	<div> <div></div> <div></div> <div></div> </div>

ID: 911c415b

$$(7532 + 100y^2) + 10(10y^2 - 110)$$

The expression above can be written in the form $ay^2 + b$, where a and b are constants. What is the value of $a + b$?

ID: 911c415b Answer

Rationale

The correct answer is 6632. Applying the distributive property to the expression yields $(7532 + 100y^2) + (100y^2 - 1100)$. Then adding together $7532 + 100y^2$ and $100y^2 - 1100$ and collecting like terms results in $200y^2 + 6432$. This is written in the form $ay^2 + b$, where $a = 200$ and $b = 6432$. Therefore $a + b = 200 + 6432 = 6632$.

Question Difficulty: Hard

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Equivalent expressions	<div><div></div><div></div><div></div></div>

ID: f89e1d6f

If $a = c + d$, which of the following is equivalent to the expression $x^2 - c^2 - 2cd - d^2$?

- A. $(x + a)^2$
- B. $(x - a)^2$
- C. $(x + a)(x - a)$
- D. $x^2 - ax - a^2$

ID: f89e1d6f Answer

Correct Answer: C

Rationale

Choice C is correct. Factoring -1 from the second, third, and fourth terms gives $x^2 - c^2 - 2cd - d^2 = x^2 - (c^2 + 2cd + d^2)$. The expression $c^2 + 2cd + d^2$ is the expanded form of a perfect square: $c^2 + 2cd + d^2 = (c + d)^2$. Therefore, $x^2 - (c^2 + 2cd + d^2) = x^2 - (c + d)^2$. Since $a = c + d$, $x^2 - (c + d)^2 = x^2 - a^2$. Finally, because $x^2 - a^2$ is the difference of squares, it can be expanded as $x^2 - a^2 = (x + a)(x - a)$.

Choices A and B are incorrect and may be the result of making an error in factoring the difference of squares $x^2 - a^2$. Choice D is incorrect and may be the result of incorrectly combining terms.

Question Difficulty: Hard

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Equivalent expressions	■ ■ ■

ID: e117d3b8

If a and c are positive numbers, which of the following is equivalent to $\sqrt{(a+c)^3} \cdot \sqrt{a+c}$?

- A. $a + c$
- B. $a^2 + c^2$
- C. $a^2 + 2ac + c^2$
- D. a^2c^2

ID: e117d3b8 Answer

Correct Answer: C

Rationale

Choice C is correct. Using the property that $\sqrt{x}\sqrt{y} = \sqrt{xy}$ for positive numbers x and y , with $x = (a+c)^3$ and $y = a+c$, it follows that $\sqrt{(a+c)^3} \cdot \sqrt{a+c} = \sqrt{(a+c)^4}$. By rewriting $(a+c)^4$ as $((a+c)^2)^2$, it is possible to simplify the square root expression as follows: $\sqrt{((a+c)^2)^2} = (a+c)^2 = a^2 + 2ac + c^2$.

Choice A is incorrect and may be the result of $\sqrt{(a+c)^3} \div \sqrt{(a+c)}$. Choice B is incorrect and may be the result of incorrectly rewriting $(a+c)^2$ as $a^2 + c^2$. Choice D is incorrect and may be the result of incorrectly applying properties of exponents.

Question Difficulty: Hard